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# USE OF THE MILITARY STANDARD PLANS FOR HAZARD RATE UNDER THE WEIBULL DISTRIBUTION*

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## Summary

This paper presents a procedure, together with necessary tables of products, for applying the MIL-STD-105B plans to acceptance sampling inspection when the quality of items in the lot is evaluated in terms of the instantaneous failure rate or hazard rate as a function of time. Inspection of sample items is by attributes with life testing truncated at the end of some pre-assigned period. The Weibull distribution (including the exponential as a special case) is assumed as the underlying life-length model. Examples of application are given.


## Introduction

The procedure and tables presented in this paper are based on recent work reported on in some detail in a paper to be presented by the authors at the Eighth National Symposium on Reliability and Quality Control(1). The objective of this procedure and set of tables is to provide a simple means of adapting the MIL-STD-105B(2) plans to reliability and life-testing applications for which the lot quality of interest is the hazard rate or instantaneous failure rate for the items at some specified time or life and for which the Weibull distribution (including the exponential as a special case) can be assumed. Both the procedure and tables are of a form similar to that for the procedure and tables prepared by the authors for use when lot quality is evaluated in terms of mean item life and which were presented at the Fifteenth Annual Convention of the American Society for Quality Control(3). The two sets of tables thus supplement each other, and since the sampling inspection procedures for both are identical, they provide a comprehensive collection offering alternative criteria for lot evaluation. Additional information on Weibull sampling plans, on estimation of the Weibull parameters, and on the mathematics involved may be found in other papers by the authors[(4),(5)].

## The Acceptance-Sampling Procedure

The acceptance-sampling procedure to be employed with the tables presented here follows the usual pattern for attribute sampling inspection. The single variation in procedure is the testing of sample items for life (rather than for some other item quality characteristic) and the use of a pre-assigned test truncation time, t. For single-sampling, the following are the steps required:

(a) Making use of the tables of factors provided, select a suitable sampling-inspection plan.

(b) Draw at random a sample of items of size n as specified by the plan.

(c) Place the sample items on life test for the specified period of time, t.

(d) Determine the number of sample items that fail during the test period.

(e) Compare the number of items that fail with the acceptance number, c, specified for the plan.

(f) If the number that fail is equal to or less than the acceptance number, accept the lot; if the number failing exceeds the acceptance number, reject the lot.

Both the sample sizes and the acceptance numbers used will be those specified for the MIL-STD-105B plans. Single sampling will presumably be used, but by simple modification double or multiple sampling can be used if desired. The selection of a suitable plan, the determination of operating characteristics in terms of hazard rate for a specified plan, and the determination of an appropriate life-testing time are all made through use of the tables of products which will be found at the end of this report. Ways for making use of these tables will be discussed in the section that follows and in the accompanying illustrative examples.

It may be noted that the probability of acceptance for a lot under the procedure outlined above depends only on the probability, which may be designated by $p'$, of an item failing before the end of the test period, $t$. The actual life at which an item fails need not be determined; inspection is on an attribute basis. For this reason it is possible to use the 105B plans to evaluate submitted lots in terms of hazard rate, $Z(t)$, at some specified time, $t$. The operating characteristics for any sampling plan specified by c and n depend only on $t$ and $Z(t)$. A brief outline of the mathematics involved in the procedure will be found in the Appendix. Additional details may be found in the paper of the authors first cited[1].

To provide a procedure of simple form and one suitable for general use, the tables of factors for adapting the 105B plans to use in terms of hazard rates have been prepared in terms of dimensionless products of $t$ times $Z(t)$. Actually, in order to give figures that may be more conveniently used, the tables are composed of $tZ(t) \times 100$ products. Each of the 105B plans is cataloged in terms of such $tZ(t) \times 100$ products. These product values are to be used in much the same way that percent defective values are used in the selection and application of plans for ordinary attribute inspection. With $t$ and $Z(t)$ specified, all that must be done is to compute their product and then select a plan in its terms. If, on the other hand, a plan (n and c) and a test truncation time ($t$) are specified, the product may be used to give an evaluation of operating characteristics in terms of hazard rate, $Z(t)$. Or, alternatively, the product may be used to find a suitable test truncation time, $t$. Examples of such application of the factors will be found in the latter part of this report.

The Weibull distribution, one will recall, is a three-parameter distribution, requiring a location or threshold parameter, a scale parameter, and a shape parameter for complete description. For the procedure and plans presented here, the scale parameter need not be ascertained or known; the $tZ(t) \times 100$ product contains information on its magnitude. For the location or threshold parameter (commonly symbolized by the letter $\gamma$), a value of zero is to be assumed in the direct application of the products and the procedure. For many applications a $\gamma$ value of 0 will apply; there will be no initial period of life free of risk of failure. This assumption is equivalent to knowing the value for gamma. If $\gamma$ has some non-zero value, all that is necessary is to subtract the value for $\gamma$ from $t$ to obtain a value $t_0$ and then use $t_0$ rather than $t$ in working with the tables of products. The third parameter, the Weibull shape parameter (which is commonly symbolized by the letter $\beta$) must be known. The products depend on the value for $\beta$ so that its magnitude must be known or estimated from past research, engineering, or inspection data. Separate tables of products have been provided for each of eleven values for $\beta$, values ranging from 1/3 to 5. The assumed value for $\beta=1$ represents the special case of the exponential distribution and so may be used when this distribution seems to be the most appropriate statistical model. Estimation procedures for $\beta$ or $\gamma$ are available and may be found in the paper by Kao[4],[6].

One final point of procedure must be mentioned. It is that for the direct use of the tables of products, the item life at which the hazard rate is measured or specified and the life at which the testing of sample items is truncated are assumed to be the same. That is, the $t$ used in the $tZ(t) \times 100$ products is the same as the test truncation time, $t$. However, if the life at which hazard rates are to be specified or evaluated must differ from the test truncation time, a simple variation in procedure may be made to allow for any desired departure. All that must be done is to find (for the value for $\beta$ assumed) the hazard rate at the test truncation time that corresponds to the hazard rate at the time used in the specification or at which lots are to be evaluated. A table of hazard rate ratios has been compiled for this purpose. It will be found as Table 2 at the end of this report. The method for its use will be described in one of the illustrative examples which will follow.

## The Tables of Products

To provide the necessary data for using the 105B plans in life and reliability testing in terms of hazard rate, eleven tables of $tZ(t) \times 100$ products have been prepared. One is available for each of the following values for $\beta$: $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, 4, and 5. These values encompass the range of shape parameter values that will commonly be encountered. For values of interest for which no table has been provided (but within the above range), linear interpolation will give factors accurate enough for most practical purposes. The tables of factors have been included at the end of this report as Tables 1-A through 1-K.

Each table provides $tZ(t) \times 100$ products for each 105B plan, that is for each combination of Sample Size Code Letter and Acceptable Quality Level the 105B Standard utilizes. The factors apply not only to the single-sampling plans, but also to the matched double-sampling and multiple-sampling plans. The sample sizes corresponding to each Sample Size Code Letter and the acceptance and rejection numbers applying to each Acceptable Quality

Level as found in the 105B Standard are thus to be used in the procedure presented in this paper.

Across the top of each table will be found $tZ(t) \times 100$ products corresponding to each of the Acceptable Quality Level values utilized in the 105B tables. Each of these matched $tZ(t) \times 100$ products provides for all of the 105B plans of the corresponding Acceptable Quality Level a measure of lot quality for which the probability of acceptance is high. The producer's risk will be low with its actual magnitude being the same as that experienced under normal use of the 105B plans. It will be recalled that this risk varies, being as low as .01 for large sample sizes and as high as .20 for small sample sizes. The risk associated with a specific plan may be obtained from the operating characteristic curves provided in the 105B standard.

Within the body of each table will be found $tZ(t) \times 100$ products for each plan for which the probability of acceptance is low. In each case this probability is .10. These products may be used as a means of estimating the consumer's risk when a Rejectable Hazard Rate is to be determined or when a plan is to be selected in such terms. Thus with pairs of factors provided for use in terms of both an Acceptable Hazard Rate as found across the top of each table and a Rejectable Hazard Rate as found within the body of each table, plans may be selected or evaluated in terms of either the producer's risk or the consumer's risk or both. Recently a proposed revision "C" of MIL-STD-105 included, among other things, the listing of LTPD values as consumer's protection measures as well as the AQL values.[7]

The interpretation of these $tZ(t) \times 100$ products may be demonstrated by means of a simple example. Suppose that for a product to be submitted to acceptance inspection $\beta$ can be assumed to have a value of $\frac{1}{2}$ and $\gamma$ a value of 0. Life testing of sample items is to be truncated at 500 hours. A single-sampling plan for Sample Size Code Letter H and an AQL of 1.5% has been specified for use (n, the sample, size will thus be 35 and c, the acceptance number, 1). Reference to Table 1-B which contains products for $\beta = \frac{1}{2}$ shows that the $tZ(t) \times 100$ product at the Acceptable Quality Level of 1.5% is .756. With $t = 500$ thus $500Z(t) \times 100 = .756$. $Z(t)$, the hazard rate, can thus be computed as $Z(t) = .756/500 \times 100 = .0000151$ (per hour). Accordingly, this figure, .0000151, can be considered as the Acceptable Hazard Rate (as measured at 500 hours of life). Next, by entering the body of the table at Sample Size Code Letter H and an AQL of 1.5%, the $tZ(t) \times 100$ product for which the probability of acceptance is low (.10) is 5.5. Again with $t = 500$, $500Z(t) \times 100 = 5.5$ or $Z(t) = 5.5/500 \times 100 = .00011$. Thus the Rejectable Hazard Rate (at 500 hours of life) is .00011 (per hour). These two figures found for $Z(t)$ may be interpreted thus; (a) lots for which the hazard rate for items at 500 hours of life is .0000151 per hour or less will have a high probability of acceptance (examination of the OC curves in the 105B Standards indicates the probability is slightly more than .90); lots for which the hazard rate for items at 500 hours of life is .00011 will have a low probability of acceptance (namely .10 or less).

## Examples

Example (1):

A purchased electronic component is to be inspected, lot by lot, for lifelength by sampling inspection. The MIL-STD-105B plans are to be employed. The lot quality of interest is the hazard rate at a life of 500 hours. From past experience with the component it has been determined that the Weibull distribution can be applied as a statistical model and that a value for $\beta$, the shape parameter, of $\frac{2}{3}$ can be assumed and a value for $\gamma$, the location or threshold parameter, of 0 can be expected. Normal inspection is to be employed and the lot size in each case will be 3,000 items. A plan employing an Acceptable Quality Level (in terms of the percent defective as used in the standard) of 4.0% and a test period of 500 hours have been tentatively selected, with single sampling to be utilized. Under these given conditions, the acceptance procedure and the operating characteristics in terms of hazard rate at 500 hours are to be determined.

By referring to Table III of MIL-STD-105B it will be found that for Inspection Level II (whi h is customarily employed unless there are special reasons for doing otherwise) and for a lot size of 3,000, Sample Size Code Letter L should be employed. Next, reference to Table IV-A of the same Military Standard shows that for Sample Size Code Letter L the sample size for single sampling is 150 items. Further reference to this latter Table shows that for the 4.0% Acceptable Quality Level selected the acceptance number to use is 11 and the rejection number 12. The acceptance-rejection procedure will accordingly be:

(a) Draw at random from the lot a sample of 150 items.

(b) Place the sample items on life test for 500 hours.

(c) Determine the number of sample items that fail prior to the end of this test period.

(d) If the number that fail is 11 or less, accept the lot; if the number that fail is 12 or more, reject it.

The operating characteristics of the above acceptance procedure in terms of hazard rate can be determined by reference to Table I-C which will be found at the end of this report. This table lists $tZ(t) \times 100$ products for application when $\beta = \frac{2}{3}$. Examination of the factors across the top of the table (immediately below the AQL figures in $p'(\%)$) shows that at 4.0% the corresponding $tZ(t) \times 100$ ratio is 2.72. With $t = 500$ and with $tZ(t) \times 100 = 2.72$, the hazard rate is thus determined by:

$$500Z(t) \times 100 = 2.72$$

$$Z(t) = \frac{2.72}{500 \times 100} = .0000544 \quad .$$

This figure may be considered as an Acceptable Hazard Rate (AHR). Thus if the quality of the submitted lot is such that the hazard rate at 500 hours is .0000544 (per hour), the probability of its acceptance will be high. By reference to Table VI-L of the Military Standard, operating characteristic curves for single sampling for Code Letter L may be found. The curve for a 4.0% AQL indicates that at 4.0% the probability of acceptance is slightly more than .98; the producer's risk, then, for lots for which the hazard rate is .0000544 at 500 hours is 1.00-.98 or .02.

A Rejectable Hazard Rate (RHR), a rate at which the probability of acceptance will be low under the above plan, may also be determined by further reference to Table I-C of this report. In the body of this table at Sample Size Code Letter L and an AQL of 4.0%, a second $tZ(t) \times 100$ product for the application, one whose value is 7.9, will be found. This is the product for which the probability of acceptance is .10 or less. Using $t = 500$ and making another simple computation gives:

$$500Z(t) \times 100 = 7.9$$

$$Z(t) = \frac{7.9}{500 \times 100} = .00016 \quad .$$

This answer may be considered as a Rejectable Hazard Rate (RHR). If the lot quality is such that the hazard rate is .00016 (per hour) at 500 hours of life, the probability of acceptance for the lot will be low, namely .10 or less. This figure quantifies the consumer's risk at the Rejectable Hazard Rate of .00016.

Example (2):

For another illustration of use, consider a sampling inspection application for which an Acceptable Hazard Rate of .000025 (per hour) and a Rejectable Hazard Rate of .00015 (per hour) with both at 1000 hours of life is required. A value for $\beta$ of $1\frac{2}{3}$ and for $\gamma$ of 0 can be assumed. The test time for sample items is to be 1,000 hours with double sampling to be employed. The problem is to select from the MIL-STD-105B plans the one that will meet most closely these requirements.

Computations at the Acceptable Hazard Rate give $tZ(t) \times 100 = 1{,}000 \times .000025 \times 100 = 2.5$. Reference may now be made to Table I-F of this report which lists $tZ(t) \times 100$ products for application when $\beta = 1\frac{2}{3}$. In the line of factors across the top of the table (the column headings) the factor 2.52 will be found corresponding to a 1.5% AQL. Use of any plan with this AQL percentage will thus meet closely the Acceptable Hazard Rate requirement; the probability of acceptance for lots meeting this specified figure will be high.

Computations at the Rejectable Hazard Rate give $tZ(t) \times 100 = 1{,}000 \times .00015 \times 100 = 15$. Examination of the column of products for the 1.5% AQL figure shows that for Sample Size Code Letter J the $tZ(t) \times 100$ product is 15. Thus use of this Code Letter (with the AQL value selected) will provide a low probability of acceptance (.10 or less) for lots at which the hazard rate is .00015 at 1000 hours. Any 105B plan (single, double, or multiple) with a 1.5 AQL and a Sample Size Code Letter J will meet closely the operational requirements.

For double sampling, reference to Table IV-B of the 105B Standard indicates the first sample size is 50 and the second sample size is 100 for Letter J. At the AQL value of 1.5 the acceptance number under normal inspection for the first sample must be 1 and the rejection number 6; for combined samples the acceptance number must be 5 and the rejection number 6. Other details of the acceptance-rejection procedure will be those customarily employed in double sampling. It should be noted that the life testing time for the first sample must be 1,000 hours; likewise, it must be 1,000 hours for the second sample when testing of a second sample becomes necessary.

Example (3):

Consider a case for which a $\beta$ value of $2\frac{1}{2}$ and a $\gamma$ value of 0 can be assumed. A Rejectable Hazard Rate of .00090 (per hour) at a life of 2,000 hours has been specified. However, a test period of only 500 hours for sample items is, for practical reasons, the longest that can be utilized. The problem is to determine a single-sampling 105B plan that will meet the RHR requirement.

A suitable plan in terms of this required time for the lot quality specification but one that uses the reduced testing time can be found through application of data in Table 2 of this report, a table of hazard rate ratios. If we let $t_2$ represent the life at which the hazard rate is specified and $t_1$ represent the testing time to be employed, then the ratio between the two which is required for the use of the table may be determined. It is $t_2/t_1$ or 2000/500 which is 4. Entering the table at the 4.00 value for this ratio and reading across to values for use when $\beta = 2\frac{1}{2}$, a $Z(t_2)/Z(t_1)$ ratio of 8.00 may be found. Letting $Z(t_2)$ represent the hazard rate specified at time $t_2$ of 2,000 hours (which is .00090), a corresponding hazard rate $Z(t_1)$ at time $t_1$ of 500 hours can be computed. Since $Z(t_2)/Z(t_1) = 8.00$, $Z(t_1) = .00090/8.00 = .00011$. A $tZ(t) \times 100$ ratio can now be computed using this new rate to apply at 500 hours. It is $500 \times .00011 \times 100 = 5.5$

With this value for the RHR product, 5.5, one may now make reference to

Table I-H which lists products for $\beta = 2\frac{1}{2}$. Any plan for which a value at or close to 5.5 may be found in the body of this table will meet the RHR specifications. For example, a plan with an AQL of 0.25% and with Sample Size Code Letter N will do; likewise, one with an AQL of 1.0% and with Sample Size Code Letter Q meets the requirement. A choice from these or other suitable alternatives will depend on the Acceptable Hazard Rate that seems most suitable for the case. Use of the $tZ(t) \times 100$ product at the top of a column from which a plan is selected will enable one to determine the hazard rate for which the probability of acceptance will be high. For the first alternative mentioned above (0.25% and N), $tZ(t) \times 100 = .625$. Thus the AHR $= .625/500 \times 100 = .0000125$ at 500 hours. At 2000 hours it depends on the ratio $Z(t_2)/Z(t_1) = 8.00$. Thus at 2000 hours the AHR, $Z(t_2) = .0000125 \times 8.00 = .00010$.

Example (4):

For an example of a case for which the location or threshold parameter is not zero, consider an application for which one can assume $\beta = 4$ and $\gamma = 200$ hours. A sample size of 50 has been specified (single sampling is to be employed). A life testing time of 600 hours for sample items has been agreed upon. It seems reasonable to expect a hazard rate of .00070 at 600 hours and so this figure is selected as an Acceptable Hazard Rate and a plan is accordingly to be selected in these terms. The problem is to determine what acceptance number must be used and also what measure of consumer protection will be afforded.

The first step is to subtract the value for $\gamma$ from the value for $t$, the test and specification time value, to get $t_o$, a value in zero threshold-parameter terms. Accordingly, $t - \gamma = 600 - 200 = 400 = t_o$. This converted value may now be used in the normal manner for the tables and procedures being described in this report.

The $t_oZ(t) \times 100$ product at the Acceptable Hazard Rate will thus be $400 \times .00070 \times 100$ or 28. Reference may now be made to Table I-J which lists products for $\beta = 4$. Examination of the column headings indicates for an AQL of 6.5% a corresponding $tZ(t) \times 100$ product of 26.9, a figure reasonably close to 28. Reference next to table IV-A of MIL-STD-105B shows that for single sampling with a sample of size 50, and with an AQL of 6.5%, the acceptance number must be 6. Also, it may be noted that a single sample of size 50 corresponds to Sample Size Code Letter I.

To determine a measure of consumer protection when the sample size is 50 (Letter I) and the acceptance number is 6 (AQL = 6.5%), reference may now be made again to Table I-J of this report. At Letter I and an AQL of 6.5%, it will be found that the $tZ(t) \times 100$ product is 89 at the Rejectable Hazard Rate (for which the probability of acceptance is low, namely .10 or less). With this value one may determine that $t_oZ(t) \times 100 = 400\ Z(t) \times 100 = 89$, or $Z(t) = 89/400 \times 100 = .0022$. Note again for these computations the converted time value, $t_o$, is used. The Rejectable Hazard Rate of .0022 as computed above actually applies, however, at a real life of $(t_o + \gamma)$ or 600 hours.

## Appendix

### The Relationship Between p' and tZ(t)

The hazard rate at a specified time t, Z(t), may be expressed by the equation

$$Z(t) = f(t) \,/\, [1 - F(t)] \tag{1}$$

for which f(t) is the population density function and F(t) is the cumulative distribution function.

For the Weibull case (letting $\gamma$, the location or threshold parameter equal 0), the following relationships held,

$$f(t) = (\beta/\eta)\ (t/\eta)^{\beta-1}\ \exp[-(t/\eta)^{\beta}] \quad , \tag{2}$$

and

$$F(t) = 1 - \exp[-(t/\eta)^{\beta}] \tag{3}$$

where $\eta$ is the scale parameter and $\beta$ the shape parameter.

Substitution of the expression for f(t) given by Equation 2 and for F(t) by Equation 3 into Equation 1 and simplifying gives

$$Z(t) = (\beta/\eta)\ (t/\eta)^{\beta-1} \quad . \tag{4}$$

Multiplying each side of Equation 4 by $(t/\beta)$ gives the equation

$$\frac{tZ(t)}{\beta} = (t/\eta)^{\beta} \quad . \tag{5}$$

The probability, p', of an item failing before time t is given by the cumulative distribution function.

$$p' = F(t) = 1 - \exp[-(t/\eta)^{\beta}] \tag{6}$$

By combining Equations 5 and 6, p' in terms of tZ(t) is thus found to be

$$p' = 1 - \exp[-\frac{tZ(t)}{\beta}] \quad , \text{ or} \tag{7}$$

$$tZ(t) = -\beta\, \ln(1-p') \quad . \tag{8}$$

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TABLE 1-A

Table of tZ(t) x 100 Products for β =1/3

| Sample Size Code Letter | Acceptable Quality Level (AQL) | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | p'(%) | 0.015 | 0.035 | 0.065 | 0.10 | 0.15 | 0.25 | 0.40 | 0.65 | 1.0 | 1.5 | 2.5 | 4.0 | 6.5 | 10.0 |
| | tZ(t) x 100 | 005 | .012 | .022 | .033 | .050 | .083 | .134 | .217 | .335 | .504 | .844 | 1.36 | 2.24 | 3.51 |
| A | tZ(t) x 100 Product at RHR [P(A) = .10] | | | | | | | | | | | | | ↓ | 38 |
| B | | | | | | | | | | | | | ↓ | 24 | ↑ |
| C | | | | | | | | | | | | ↓ | 15 | ↑ | ↓ |
| D | | | | | | | | | | | ↓ | 10 | ↑ | ↓ | 19 |
| E | | | | | | | | | | ↓ | 7.6 | ↑ | ↓ | 13 | 19 |
| F | | | | | | | | | ↓ | 5.0 | ↑ | ↓ | 8.9 | 12 | 16 |
| G | | | | | | | | ↓ | 3.0 | ↑ | ↓ | 5.3 | 7.3 | 9.6 | 13 |
| H | | | | | | | ↓ | 2.1 | ↑ | ↓ | 3.7 | 5.2 | 6.6 | 9.6 | 12 |
| I | | | | | | ↓ | 1.5 | ↑ | ↓ | 2.5 | 3.6 | 4.4 | 5.6 | 7.4 | 10 |
| J | | | | | ↓ | 1.0 | ↑ | ↓ | 1.7 | 2.3 | 3.0 | 3.6 | 4.9 | 6.7 | 9.7 |
| K | | | | ↓ | .69 | ↑ | ↓ | 1.1 | 1.6 | 2.0 | 2.4 | 3.2 | 4.0 | 5.9 | 8.4 |
| L | | | ↓ | .50 | ↑ | ↓ | .85 | 1.1 | 1.5 | 1.8 | 2.0 | 2.9 | 3.9 | 5.6 | 7.8 |
| M | | ↓ | .36 | ↑ | ↓ | .57 | .79 | .99 | 1.2 | 1.4 | 1.9 | 2.5 | 3.7 | 5.0 | 7.0 |
| N | | .25 | ↑ | ↓ | .43 | .59 | .73 | .88 | 1.0 | 1.3 | 1.7 | 2.3 | 3.1 | 4.9 | 6.6 |
| O | | ↑ | ↓ | .28 | .39 | .49 | .59 | .68 | .87 | 1.1 | 1.5 | 2.0 | 2.8 | 4.1 | 5.9 |
| P | | ↓ | .17 | .23 | .29 | .35 | .46 | .57 | .73 | .94 | 1.2 | 1.8 | 2.5 | 3.7 | 5.3 |
| Q | | .086 | .11 | .15 | .20 | .26 | .31 | .42 | .55 | .72 | .97 | 1.5 | 2.1 | 3.2 | 4.9 |

TABLE 1-B

Table of tZ(t) x 100 Products for β = 1/2

| Sample Size Code Letter | Acceptable Quality Level (AQL) | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | p'(%) | 0.015 | 0.035 | 0,065 | 0.10 | 0.15 | 0.25 | 0.40 | 0.65 | 1,0 | 1.5 | 2.5 | 4.0 | 6.5 | 10.0 |
| | tZ(t) x 100 | .007 | .017 | .032 | .050 | .075 | .125 | .201 | .326 | .503 | .756 | 1.27 | 2.04 | 3.36 | 5.27 |
| A | tZ(t) x 100 Product at RHR [P(A) = .10] | | | | | | | | | | | | | ↓ | 56 |
| B | | | | | | | | | | | | | ↓ | 34 | ↑ |
| C | | | | | | | | | | | | ↓ | 22 | ↑ | ↓ |
| D | | | | | | | | | | | ↓ | 16 | ↑ | ↓ | 29 |
| E | | | | | | | | | | ↓ | 11 | ↑ | ↓ | 20 | 29 |
| F | | | | | | | | | ↓ | 7.6 | ↑ | ↓ | 13 | 18 | 25 |
| G | | | | | | | | ↓ | 4.5 | ↑ | ↓ | 7.9 | 10 | 14 | 20 |
| H | | | | | | | ↓ | 3.2 | ↑ | ↓ | 5.5 | 7.8 | 10 | 14 | 18 |
| I | | | | | | ↓ | 2.2 | ↑ | ↓ | 3.8 | 5.4 | 6.9 | 8.2 | 11 | 15 |
| J | | | | | ↓ | 1.5 | ↑ | ↓ | 2.6 | 3.5 | 4.4 | 5.4 | 7.2 | 10 | 14 |
| K | | | | ↓ | 1.0 | ↑ | ↓ | 1.7 | 2.4 | 3.0 | 3.6 | 4.8 | 6.0 | 8.8 | 12 |
| L | | | ↓ | .76 | ↑ | ↓ | 1.2 | 1.7 | 2.2 | 2.7 | 3.1 | 4.2 | 5.8 | 8.3 | 11 |
| M | | ↓ | .51 | ↑ | ↓ | .86 | 1.1 | 1.4 | 1.8 | 2.1 | 2.9 | 3.7 | 5.5 | 7.4 | 10 |
| N | | .38 | ↑ | ↓ | .65 | .88 | 1.1 | 1.3 | 1.5 | 1.9 | 2.6 | 3.4 | 4.7 | 7.2 | 9.8 |
| O | | ↑ | ↓ | .43 | .59 | .74 | .88 | 1.0 | 1.3 | 1.7 | 2.2 | 3.1 | 4.1 | 6.0 | 8.8 |
| P | | ↓ | .26 | .35 | .49 | .53 | .70 | .86 | 1.1 | 1.4 | 1.8 | 2.7 | 3.7 | 5.5 | 8.0 |
| Q | | .13 | .17 | .22 | .31 | .39 | .47 | .63 | .82 | 1.0 | 1.4 | 2.2 | 3.2 | 4.8 | 7.2 |

TABLE 1-C

Table of tZ(t) x 100 Products for β = 2/3

| Sample Size Code letter | | Acceptable Quality Level (AQL) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | p'(%) | 0.015 | 0.035 | 0.065 | 0.10 | 0.15 | 0.25 | 0.40 | 0.65 | 1.0 | 1.5 | 2.5 | 4.0 | 6.5 | 10.0 |
| | tZ(t) x 100 | .010 | .023 | .043 | .067 | .100 | .167 | .267 | .435 | .670 | 1.01 | 1.69 | 2.72 | 4.48 | 7.02 |
| A | tZ(t) x 100 Product at RHR [P(A) = .10] | | | | | | | | | | | | | ↓ | 76 |
| B | | | | | | | | | | | | | ↓ | 50 | ↑ |
| C | | | | | | | | | | | | ↓ | 30 | ↑ | ↓ |
| D | | | | | | | | | | | ↓ | 21 | ↑ | ↓ | 39 |
| E | | | | | | | | | | ↓ | 15 | ↑ | ↓ | 27 | 39 |
| F | | | | | | | | | ↓ | 10 | ↑ | ↓ | 17 | 25 | 33 |
| G | | | | | | | | ↓ | 6.0 | ↑ | ↓ | 10 | 14 | 18 | 27 |
| H | | | | | | | ↓ | 4.3 | ↑ | ↓ | 7.4 | 10 | 13 | 18 | 24 |
| I | | | | | | ↓ | 3.0 | ↑ | ↓ | 5.1 | 7.2 | 9.2 | 11 | 15 | 20 |
| J | | | | | ↓ | 2.0 | ↑ | ↓ | 3.4 | 4.7 | 6.0 | 7.2 | 9.6 | 13 | 19 |
| K | | | | ↓ | 1.3 | ↑ | ↓ | 2.3 | 3.2 | 4.0 | 4.9 | 6.5 | 8.0 | 11 | 16 |
| L | | | ↓ | 1.0 | ↑ | ↓ | 1.7 | 2.3 | 3.0 | 3.5 | 4.1 | 5.8 | 7.9 | 11 | 15 |
| M | | ↓ | .68 | ↑ | ↓ | 1.1 | 1.5 | 1.9 | 2.3 | 2.7 | 3.9 | 5.0 | 7.4 | 10 | 14 |
| N | | .51 | ↑ | ↓ | .86 | 1.1 | 1.5 | 1.7 | 2.0 | 2.6 | 3.4 | 4.6 | 6.2 | 9.6 | 13 |
| O | | ↑ | ↓ | .57 | .78 | .98 | 1.1 | 1.3 | 1.7 | 2.2 | 3.0 | 4.1 | 5.6 | 8.2 | 11 |
| P | | ↓ | .34 | .47 | .59 | .71 | .93 | 1.1 | 1.5 | 1.9 | 2.4 | 3.5 | 5.0 | 7.4 | 10 |
| Q | | .17 | .23 | .30 | .41 | .52 | 63 | .84 | 1.1 | 1.4 | 1.9 | 3.0 | 4.2 | 6.5 | 9.6 |

TABLE 1-D

Table of tZ(t) x 100 Products for β = 1

| Sample Size Code Letter | | Acceptable Quality Level (AQL) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | p'(%) | 0.015 | 0.035 | 0.065 | 0.10 | 0.15 | 0.25 | 0.40 | 0.65 | 1.0 | 1.5 | 2.5 | 4.0 | 6.5 | 10.0 |
| | tZ(t) x 100 | .015 | .035 | .065 | .100 | .150 | .250 | .401 | .652 | 1.01 | 1.51 | 2.53 | 4.08 | 6.72 | 10.5 |
| A | tZ(t) x 100 Product at RHR [P(A) = .10] | | | | | | | | | | | | | ↓ | 115 |
| B | | | | | | | | | | | | | ↓ | 77 | ↑ |
| C | | | | | | | | | | | | ↓ | 46 ↑ | ↑ | ↓ |
| D | | | | | | | | | | | ↓ | 33 | | ↓ | 61 |
| E | | | | | | | | | | ↓ | 23 | ↑ | ↓ | 41 | 60 |
| F | | | | | | | | | | 15 | ↑ | | 27 | 38 | 50 |
| G | | | | | | | | | ↓ 9.2 | ↑ | ↓ | ↓ 16 | 22 | 28 | 42 |
| H | | | | | | | ↓ | ↓ 6.5 | ↑ | ↓ | 11 | 16 | 20 | 28 | 38 |
| I | | | | | | | 4.6 ↑ | ↑ | ↓ | 7.8 | 11 | 14 | 17 | 23 | 31 |
| J | | | | | ↓ | ↓ 3.1 | | ↓ | 5.3 | 7.2 | 9.1 | 11 | 15 | 20 | 29 |
| K | | | | ↓ | 2.1 | ↑ | ↓ | 3.6 | 4.9 | 6.2 | 7.4 | 9.9 | 12 | 18 | 26 |
| L | | | | 1.5 | ↑ | | 2.6 | 3.6 | 4.5 | 5.4 | 6.3 | 9.0 | 12 | 17 | 24 |
| M | | ↓ | ↓ 1.0 | ↑ | ↓ | ↓ 1.8 | 2.4 | 3.0 | 3.6 | 4.2 | 6.0 | 7.6 | 11 | 15 | 21 |
| N | | .78 | ↑ | ↓ | 1.3 | 1.8 | 2.3 | 2.7 | 3.1 | 4.0 | 5.3 | 6.9 | 9.5 | 14 | 20 |
| O | | ↑ | ↓ | .86 | 1.2 | 1.5 | 1.8 | 2.1 | 2.7 | 3.5 | 4.5 | 6.2 | 8.7 | 13 | 18 |
| P | | ↓ | .52 | .72 | .91 | 1.1 | 1.4 | 1.8 | 2.3 | 2.9 | 3.7 | 5.4 | 7.6 | 11 | 16 |
| Q | | .26 | .35 | .45 | .63 | .79 | .97 | 1.3 | 1.7 | 2.2 | 3.0 | 4.5 | 6.5 | 9.9 | 15 |

TABLE 1-E

Table of tZ(t) x 100 Products for β = 1-1/3

| Sample Size Code Letter | Acceptable Quality Level (AQL) | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | p'(%) | 0.015 | 0.035 | 0.065 | 0.10 | 0.15 | 0.25 | 0.40 | 0.65 | 1.0 | 1.5 | 2.5 | 4.0 | 6.5 | 10.0 |
| | tZ(t) x 100 | .020 | .047 | .087 | .133 | .200 | .333 | .535 | .869 | 1.34 | 2.02 | 3.38 | 5.44 | 8.96 | 14.0 |
| A | tZ(t) x 100 Product of RHR [P(A) = .10] | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 150 |
| B | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 100 | ↓ |
| C | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 61 | ↑ | ↓ |
| D | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 44 | ↑ | ↓ | 80 |
| E | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 30 | ↑ | ↓ | 55 | 80 |
| F | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 20 | ↑ | ↓ | 35 | 50 | 67 |
| G | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 12 | ↑ | ↓ | 21 | 29 | 38 | 56 |
| H | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 8.6 | ↑ | ↓ | 14 | 20 | 26 | 38 | 50 |
| I | | ↓ | ↓ | ↓ | ↓ | ↓ | 6.0 | ↑ | ↓ | 10 | 14 | 18 | 22 | 30 | 41 |
| J | | ↓ | ↓ | ↓ | ↓ | 4.0 | ↑ | ↓ | 7.0 | 9.4 | 12 | 14 | 19 | 27 | 38 |
| K | | ↓ | ↓ | ↓ | 2.7 | ↑ | ↓ | 4.7 | 6.5 | 8.2 | 9.7 | 13 | 16 | 23 | 33 |
| L | | ↓ | ↓ | 2.0 | ↑ | ↓ | 3.4 | 4.7 | 6.0 | 7.2 | 8.2 | 11 | 15 | 22 | 31 |
| M | | ↓ | 1.3 | ↑ | ↓ | 2.3 | 3.2 | 4.0 | 4.8 | 5.5 | 7.8 | 10 | 14 | 20 | 28 |
| N | | 1.0 | ↑ | ↓ | 1.7 | 2.3 | 2.9 | 3.5 | 4.1 | 5.2 | 7.0 | 9.2 | 12 | 19 | 26 |
| O | | ↑ | ↓ | 1.1 | 1.5 | 1.9 | 2.3 | 2.7 | 3.5 | 4.6 | 6.0 | 8.2 | 11 | 16 | 23 |
| P | | ↓ | .68 | .94 | 1.1 | 1.4 | 1.8 | 2.3 | 2.9 | 3.8 | 4.9 | 7.6 | 10 | 14 | 21 |
| Q | | .34 | .46 | .60 | .82 | 1.0 | 1.2 | 1.6 | 2.2 | 2.9 | 3.9 | 6.0 | 8.5 | 13 | 19 |

TABLE 1-F

Table of tZ(t) x 100 Products for β = 1-2/3

| Sample Size Code Letter | Acceptable Quality Level (AQL) | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | p'(%) | 0.015 | 0.035 | 0.065 | 0.10 | 0.15 | 0.25 | 0.40 | 0.65 | 1.0 | 1.5 | 2.5 | 4.0 | 6.5 | 10.0 |
| | tZ(t) x 100 | .025 | .058 | .108 | .167 | .250 | .417 | .668 | 1.09 | 1.68 | 2.52 | 4.22 | 6.80 | 11.2 | 17.6 |
| A | tZ(t) x 100 Product at RHR [P(A) = 10] | — | — | — | — | — | — | — | — | — | — | — | — | ↓ | 190 |
| B | | — | — | — | — | — | — | — | — | — | — | — | ↓ | 120 | ↑ |
| C | | — | — | — | — | — | — | — | — | — | — | ↓ | 76 | ↑ | — |
| D | | — | — | — | — | — | — | — | — | — | ↓ | 54 | ↑ | ↓ | 99 |
| E | | — | — | — | — | — | — | — | — | ↓ | 38 | ↑ | ↓ | 68 | 99 |
| F | | — | — | — | — | — | — | — | ↓ | 25 | ↑ | — | 44 | 63 | 84 |
| G | | — | — | — | — | — | — | ↓ | 15 | ↑ | ↓ | 26 | 36 | 47 | 69 |
| H | | — | — | — | — | — | ↓ | 10 | ↑ | ↓ | 18 | 26 | 33 | 47 | 62 |
| I | | — | — | — | — | ↓ | 7.6 | ↑ | — | 13 | 18 | 23 | 27 | 37 | 52 |
| J | | — | — | — | ↓ | 5.0 | ↑ | — | 8.6 | 11 | 15 | 18 | 24 | 33 | 48 |
| K | | — | — | ↓ | 3.4 | ↑ | — | 5.9 | 8.0 | 10 | 12 | 16 | 20 | 29 | 42 |
| L | | | ↓ | 2.5 | ↑ | — | 4.2 | 6.0 | 7.5 | 9.0 | 10 | 14 | 19 | 28 | 39 |
| M | | ↓ | 1.7 | ↑ | ↓ | 2.8 | 4.0 | 5.0 | 6.0 | 7.0 | 9.8 | 12 | 18 | 25 | 35 |
| N | | 1 2 | ↑ | — | 2.1 | 2.9 | 3.7 | 4.4 | 5.2 | 6.6 | 8.7 | 11 | 15 | 24 | 32 |
| O | | ↑ | — | 1.4 | 1.9 | 2.4 | 2.9 | 3.4 | 4.4 | 5.8 | 7.6 | 10 | 14 | 20 | 29 |
| P | | ↓ | .86 | 1.1 | 1.4 | 1.7 | 2.3 | 2.8 | 3.7 | 4.8 | 6.1 | 9.0 | 12 | 18 | 26 |
| Q | | .43 | .58 | .75 | 1.0 | 1.3 | 1.5 | 2.1 | 2.7 | 3.6 | 4.9 | 7.6 | 10 | 16 | 24 |

TABLE 1-G

Table of tZ(t) x 100 Products for β = 2

| Sample Size Code Letter | | Acceptable Quality Level (AQL) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | p'(%) | 0.015 | 0.035 | 0.065 | 0.10 | 0.15 | 0.25 | 0.40 | 0.65 | 1.0 | 1.5 | 2.5 | 4.0 | 6.5 | 10.0 |
| | tZ(t) x 100 | .030 | .070 | .130 | .200 | .300 | .500 | .802 | 1.30 | 2.01 | 3.02 | 5.06 | 8.16 | 13.4 | 21.1 |
| A | tZ(t) × 100 Product at RHR [P(A) = .10] | | | | | | | | | | | | | ↓ | 230 |
| B | | | | | | | | | | | | | ↓ | 150 | ↑ |
| C | | | | | | | | | | | | ↓ | 92 | ↑ | ↓ |
| D | | | | | | | | | | | ↓ | 66 | ↑ | ↓ | 120 |
| E | | | | | | | | | | ↓ | 46 | ↑ | ↓ | 82 | 120 |
| F | | | | | | | | | ↓ | 30 | ↑ | ↓ | 54 | 77 | 100 |
| G | | | | | | | | ↓ | 18 | ↑ | ↓ | 32 | 44 | 56 | 84 |
| H | | | | | | | ↓ | 12 | ↑ | ↓ | 22 | 31 | 40 | 56 | 75 |
| I | | | | | | ↓ | 9.1 | ↑ | ↓ | 15 | 22 | 28 | 33 | 44 | 63 |
| J | | | | | ↓ | 6.0 | ↑ | ↓ | 10 | 14 | 18 | 22 | 29 | 40 | 58 |
| K | | | | ↓ | 4.1 | ↑ | ↓ | 7.1 | 9.8 | 12 | 14 | 20 | 24 | 35 | 51 |
| L | | | ↓ | 3.0 | ↑ | ↓ | 5.1 | 7.2 | 9.0 | 10 | 12 | 17 | 23 | 34 | 48 |
| M | | ↓ | 2.0 | ↑ | ↓ | 3.4 | 4.8 | 6.1 | 7.3 | 8.4 | 11 | 15 | 22 | 30 | 42 |
| N | | 1.5 | ↑ | ↓ | 2.6 | 3.5 | 4.4 | 5.4 | 6.3 | 8.0 | 10 | 13 | 19 | 29 | 39 |
| O | | ↑ | ↓ | 1.7 | 2.3 | 2.9 | 3.5 | 4.1 | 5.3 | 7.0 | 9.1 | 12 | 17 | 25 | 35 |
| P | | ↓ | 1.0 | 1.4 | 11.7 | 2.1 | 2.8 | 3.4 | 4.4 | 5.8 | 7.4 | 10 | 15 | 22 | 32 |
| Q | | .50 | .70 | .90 | 1.2 | 1.5 | 1.9 | 2.5 | 3.3 | 4.4 | 6.0 | 9.1 | 12 | 20 | 29 |

TABLE 1-H

Table of tZ(t) x 100 Products for $\beta = 2\text{-}1/2$

tZ(t) x 100 Product at RHR [P(A) = .10]

| Sample Size Code Letter | Acceptable Quality Level (AQL) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | p'(%) | 0.015 | 0. 035 | 0.065 | 0.10 | 0.15 | 0.25 | 0.40 | 0.65 | 1.0 | 1.5 | 2.5 | 4.0 | 6.5 | 10.0 |
| | tZ(t) x 100 | .038 | .087 | .163 | .250 | .375 | .625 | 1.00 | 1.63 | 2.51 | 3.78 | 6.33 | 7.62 | 16.8 | 26.3 |
| A | | | | | | | | | | | | | | ↓ | 290 |
| B | | | | | | | | | | | | | ↓ | 190 | ↑ |
| C | | | | | | | | | | | | ↓ | 110 | ↑ | ↓ |
| D | | | | | | | | | | | ↓ | 82 | ↑ | ↓ | 140 |
| E | | | | | | | | | | ↓ | 57 | ↑ | ↓ | 100 | 140 |
| F | | | | | | | | | ↓ | 36 | ↑ | ↓ | 67 | 95 | 120 |
| G | | | | | | | | ↓ | 23 | ↑ | ↓ | 40 | 54 | 71 | 100 |
| H | | | | | | | ↓ | 16 | ↑ | ↓ | 28 | 39 | 50 | 71 | 93 |
| I | | | | | | ↓ | 11 | ↑ | ↓ | 19 | 27 | 35 | 42 | 55 | 78 |
| J | | | | | ↓ | 7.7 | ↑ | ↓ | 13 | 17 | 22 | 27 | 36 | 50 | 73 |
| K | | | | ↓ | 5.1 | ↑ | ↓ | 8.8 | 12 | 15 | 18 | 25 | 32 | 44 | 64 |
| L | | | ↓ | 3.8 | ↑ | ↓ | 6.3 | 8.9 | 11 | 13 | 15 | 22 | 29 | 42 | 59 |
| M | | ↓ | 2.5 | ↑ | ↓ | 4.3 | 5.9 | 7.5 | 9.0 | 10 | 14 | 19 | 28 | 37 | 52 |
| N | | 1.9 | ↑ | ↓ | 3.2 | 4.4 | 5.5 | 6.6 | 7.8 | 9.9 | 13 | 17 | 23 | 36 | 49 |
| O | | ↑ | ↓ | 2.1 | 2.9 | 3.7 | 4.4 | 5.1 | 6.5 | 8.6 | 11 | 15 | 21 | 31 | 45 |
| P | | ↓ | 1.3 | 1.7 | 2.2 | 2.6 | 3.5 | 4.3 | 5.5 | 7.1 | 9.0 | 13 | 18 | 28 | 40 |
| Q | | .65 | .87 | 1.1 | 1.5 | 1.9 | 2.3 | 3.1 | 4.2 | 5.5 | 7.4 | 11 | 16 | 25 | 36 |

TABLE 1-I

Table of tZ(t) x 100 Products for $\beta$ = 3-1/3

| Sample Size Code Letter | Acceptable Quality Level (AQL) | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | p'(%) | 0.015 | 0.035 | 0.065 | 0.10 | 0.15 | 0.25 | 0.40 | 0.65 | 1.0 | 1.5 | 2.5 | 4.0 | 6.5 | 10.0 |
| | tZ(t) x 100 | .050 | .117 | .217 | .333 | .500 | .833 | 1.34 | 2.17 | 3.35 | 5.04 | 8.44 | 13.6 | 22.4 | 35.1 |
| A | tZ(t) x 100 Product at RHR [P(A) = .10] | | | | | | | | | | | | | ↓ | 380 |
| B | | | | | | | | | | | | | ↓ | 250 | ↑ |
| C | | | | | | | | | | | | ↓ | 150 | ↑ | ↓ |
| D | | | | | | | | | | | ↓ | 100 | ↑ | ↓ | 190 |
| E | | | | | | | | | | ↓ | 76 | ↑ | ↓ | 130 | 190 |
| F | | | | | | | | | ↓ | 51 | ↑ | ↓ | 88 | 120 | 160 |
| G | | | | | | | | ↓ | 30 | ↑ | ↓ | 53 | 73 | 94 | 130 |
| H | | | | | | | ↓ | 21 | ↑ | ↓ | 37 | 52 | 66 | 94 | 120 |
| I | | | | | | ↓ | 15 | ↑ | ↓ | 25 | 36 | 46 | 56 | 74 | 100 |
| J | | | | | ↓ | 10 | ↑ | ↓ | 17 | 23 | 30 | 36 | 48 | 67 | 97 |
| K | | | | ↓ | 6.9 | ↑ | ↓ | 11 | 16 | 20 | 24 | 33 | 40 | 59 | 84 |
| L | | | ↓ | 5.0 | ↑ | ↓ | 8.4 | 12 | 15 | 17 | 20 | 29 | 37 | 56 | 78 |
| M | | ↓ | 3.4 | ↑ | ↓ | 5.7 | 8.0 | 10 | 12 | 14 | 19 | 25 | 37 | 50 | 70 |
| N | | 2.5 | ↑ | ↓ | 4.3 | 5.8 | 7.4 | 8.8 | 10 | 13 | 17 | 22 | 31 | 48 | 65 |
| O | | ↑ | ↓ | 2.8 | 3.9 | 4.9 | 5.9 | 6.8 | 8.8 | 11 | 15 | 20 | 28 | 41 | 59 |
| P | | ↓ | 1.7 | 2.3 | 2.9 | 3.5 | 4.6 | 5.7 | 7.4 | 9.6 | 12 | 17 | 25 | 37 | 53 |
| Q | | .86 | 1.1 | 1.5 | 2.0 | 2.6 | 3.1 | 4.2 | 5.6 | 7.3 | 9.9 | 15 | 21 | 33 | 48 |

TABLE 1-J

Table of tZ(t) x 100 Products for β = 4

| Sample Size Code Letter | Acceptable Quality Level (AQL) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | p*(%) | 0.015 | 0.035 | 0.065 | 0.10 | 0.15 | 0.25 | 0.40 | 0.65 | 1.0 | 1.5 | 2.5 | 4.0 | 6.5 | 10.0 |
| | tZ(t) x 100 | .060 | .140 | .260 | .400 | .600 | 1.00 | 1.60 | 2.61 | 4.02 | 6.04 | 10.1 | 16.3 | 26.9 | 42.1 |
| A | tZ(t) x 100 Product at RHR [P(A) = .10] | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 470 |
| B | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 310 | ↑ |
| C | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 180 | ↑ | ↓ |
| D | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 130 | ↑ | ↓ | 230 |
| E | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 92 | ↑ | ↓ | 160 | 230 |
| F | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 62 | ↑ | ↓ | 100 | 150 | 200 |
| G | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 36 | ↑ | ↓ | 65 | 89 | 110 | 160 |
| H | | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 26 | ↑ | ↓ | 45 | 64 | 80 | 110 | 150 |
| I | | ↓ | ↓ | ↓ | ↓ | ↓ | 18 | ↑ | ↓ | 31 | 44 | 56 | 68 | 89 | 120 |
| J | | ↓ | ↓ | ↓ | ↓ | 12 | ↑ | ↓ | 21 | 28 | 36 | 44 | 58 | 81 | 110 |
| K | | ↓ | ↓ | ↓ | 8.2 | ↑ | ↓ | 14 | 19 | 25 | 29 | 40 | 49 | 71 | 100 |
| L | | ↓ | ↓ | 6.0 | ↑ | ↓ | 10 | 14 | 18 | 21 | 25 | 35 | 47 | 69 | 96 |
| M | | ↓ | 4.0 | ↑ | ↓ | 6.9 | 9.5 | 12 | 14 | 16 | 23 | 30 | 45 | 60 | 85 |
| N | | 3.0 | ↑ | ↓ | 5.2 | 7.0 | 8.8 | 10 | 12 | 15 | 21 | 27 | 38 | 58 | 79 |
| O | | ↑ | ↓ | 3.4 | 4.7 | 5.9 | 7.0 | 8.2 | 10 | 13 | 18 | 25 | 34 | 50 | 71 |
| P | | ↓ | 2.0 | 2.8 | 3.5 | 4.2 | 5.6 | 6.9 | 8.8 | 11 | 14 | 21 | 30 | 45 | 66 |
| Q | | 1.0 | 1.4 | 1.8 | 2.4 | 3.1 | 3.8 | 5.0 | 6.8 | 8.8 | 11 | 18 | 25 | 40 | 58 |

TABLE 1-K

Table of tZ(t) x 100 Products for $\beta = 5$

| Sample Size Code Letter | Acceptable Quality Level (AQL) | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | p'(%) | 0.015 | 0.035 | 0.065 | 0.10 | 0.15 | 0.25 | 0.40 | 0.65 | 1.0 | 1.5 | 2.5 | 4.0 | 6.5 | 10.0 |
| | tZ(t) x 100 | .075 | .175 | .325 | .500 | .750 | 1.25 | 2.01 | 3.26 | 5.03 | 7.56 | 12.7 | 20.4 | 33.6 | 52.7 |
| A | | | | | | | | | | | | | | | 580 |
| B | | | | | | | | | | | | | | 380 | |
| C | | | | | | | | | | | | | 220 | | |
| D | | | | | | | | | | | | 160 | | | 290 |
| E | | | | | | | | | | | 110 | | | 200 | 290 |
| F | | | | | | | | | | 76 | | | 130 | 180 | 250 |
| G | | | | | | | | | 45 | | | 80 | 110 | 140 | 200 |
| H | | | | | | | | 32 | | | 56 | 79 | 100 | 140 | 180 |
| I | | | | | | | 23 | | | 38 | 54 | 70 | 84 | 110 | 150 |
| J | | | | | | 15 | | | 26 | 35 | 45 | 55 | 72 | 100 | 140 |
| K | | | | | 10 | | | 17 | 24 | 31 | 36 | 49 | 60 | 88 | 120 |
| L | | | | 7.6 | | | 12 | 17 | 22 | 27 | 31 | 44 | 58 | 84 | 110 |
| M | | | 5.1 | | | 8.6 | 11 | 15 | 18 | 21 | 29 | 37 | 55 | 75 | 100 |
| N | | 3.8 | | | 6.5 | 8.8 | 11 | 13 | 15 | 19 | 26 | 34 | 47 | 72 | 98 |
| O | | | | 4.3 | 5.9 | 7.4 | 8.8 | 10 | 13 | 17 | 23 | 31 | 43 | 62 | 88 |
| P | | | 2.6 | 3.5 | 4.9 | 5.3 | 7.0 | 8.6 | 11 | 14 | 18 | 27 | 38 | 55 | 80 |
| Q | | 1.3 | 1.7 | 2.2 | 3.1 | 3.9 | 4.7 | 6.3 | 8.4 | 11 | 14 | 23 | 32 | 49 | 72 |

TABLE 2

Table of Hazard Rate Ratios

| $\frac{t_2}{t_1}$ | $\frac{Z(t_2)}{Z(t_1)}$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\beta$ | | | | | | | | | | |
| | 1/3 | 1/2 | 2/3 | 1 | 1-1/3 | 1-2/3 | 2 | 2-1/2 | 3-1/3 | 4 | 5 |
| 1.25 | .862 | .894 | .928 | 1.00 | 1.08 | 1.16 | 1.25 | 1.40 | 1.68 | 1.95 | 2.44 |
| 1.50 | .763 | .816 | .873 | 1.00 | 1.14 | 1.31 | 1.50 | 1.84 | 2.57 | 3.38 | 5.06 |
| 1.75 | .689 | .756 | .830 | 1.00 | 1.21 | 1.45 | 1.75 | 2.32 | 3.69 | 5.36 | 9.38 |
| 2.00 | .630 | .707 | .794 | 1.00 | 1.26 | 1.59 | 2.00 | 2.83 | 5.04 | 8.00 | 16.0 |
| 2.25 | .583 | .667 | .763 | 1.00 | 1.31 | 1.72 | 2.25 | 3.38 | 6.64 | 11.4 | 25.6 |
| 2.50 | .543 | .632 | .737 | 1.00 | 1.36 | 1.84 | 2.50 | 3.95 | 8.49 | 15.6 | 39.1 |
| 2.75 | .510 | .603 | .714 | 1.00 | 1.40 | 1.96 | 2.75 | 4.56 | 10.6 | 20.8 | 57.2 |
| 3.00 | .481 | .577 | .694 | 1.00 | 1.44 | 2.08 | 3.00 | 5.20 | 13.0 | 27.0 | 81.0 |
| 3.25 | .456 | .555 | .675 | 1.00 | 1.48 | 2.19 | 3.25 | 5.86 | 15.6 | 34.3 | 112 |
| 3.50 | .434 | .534 | .659 | 1.00 | 1.52 | 2.30 | 3.50 | 6.55 | 18.4 | 42.9 | 148 |
| 3.75 | .414 | .516 | .643 | 1.00 | 1.55 | 2.42 | 3.75 | 7.26 | 21.8 | 52.7 | 198 |
| 4.00 | .397 | .500 | .630 | 1.00 | 1.59 | 2.52 | 4.00 | 8.00 | 25.4 | 64.0 | 256 |
| 4.25 | .381 | .485 | .617 | 1.00 | 1.62 | 2.62 | 4.25 | 8.76 | 29.3 | 76.8 | 326 |
| 4.50 | .367 | .472 | .606 | 1.00 | 1.65 | 2.73 | 4.50 | 9.54 | 33.4 | 91.1 | 410 |
| 4.75 | .354 | .459 | .595 | 1.00 | 1.68 | 2.83 | 4.75 | 10.4 | 37.9 | 107 | 509 |
| 5.00 | .342 | .447 | .585 | 1.00 | 1.71 | 2.92 | 5.00 | 11.2 | 42.8 | 125 | 625 |

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